ECONOMIC RESEARCH REPORTS

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RR# 98-15 **April 1998**

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* We thank Yakov Amihud, Will Baumol, Alberto Bisin, and an associate editor for comments, and the NSF and the CV Starr Center at NYU for financial help.

ABSTRACT:

We study an monetary in which final goods sell on spot markets, while labor and dividends sell through contracts. Firms and workers confuse absolute and relative price changes: A positive price-level shock makes sellers think they are producing better goods than they really are. They split this apparent windfall with workers who get a higher real wage. Hence, unexpected inflation shifts real income from firms (the principals) to workers (the agents), and thereby lowers stock-returns. A predictable money-supply rule *strictly* Pareto-dominates random money-supply rules.

Journal of Economic Literature Classification Numbers E 43, E 51.

Keywords: Stock prices, money supply, price level

1. Introduction

Stocks are usually perceived as real assets, and, as such, they should provide a good hedge against inflation. As it turns out, however, stock-returns seem to be negatively related to inflation. This paper presents a general equilibrium model with money that explains this puzzle. The explanation goes as follows: A firm cannot observe its worker's effort. It can, at first, see only a nominal signal on the worker's output -- “sales”. Before the price-level and the money supply can be observed, the worker and firm agree to a level of real compensation that does not depend on these “delayed signals”. Therefore, to motivate a worker to exert effort, firms optimally use nominal signals to determine real wages. At this point, having observed only the nominal sales of their output, firms confuse absolute and relative price changes: A positive price-level shock makes sellers think they are producing higher quality goods than they really are. They split this apparent windfall with workers who get a higher real wage. Hence, unexpected inflation shifts real income from firms (the principals) to workers (the agents), and thereby lowers stock-returns.

Final goods sell on a spot market, while the labor market and the stock market are mediated by contracts, needed to deal with the moral hazard problem. A cornerstone assumption is that these contracts must be “renegotiation-proof,” by which we mean Pareto-optimal given the firm's and the worker's preferences at the point when they have seen the firm's sales figures but before they know the general price-level. Under any such contract, a predictable money-supply rule *strictly* Pareto-dominates random money-supply rules, a stronger welfare result that the one Lucas [17] proves.

1.1 *The negative relation between stock-returns and inflation*

In most countries, the real return on stocks is negatively correlated with the inflation rate [12], a fact that still lacks a satisfying explanation although several candidates have been proposed:

(a) *Taxes*: Feldstein [9] argued that higher inflation reduces the real value of a firm's depreciation tax shield, but French, Ruback, and Schwert [10] find that, contrary to this hypothesis, the stock-returns of firms with bigger nominal tax obligations in fact suffer less from inflation than those of the average firm. Moreover, Amihud [1] shows that the relation exists in Israel even though the tax system there is designed to neutralize the effects of inflation.

(b) *Supply shocks*: Fama [8] argued that a favorable supply shock raise future expected dividends but lower the current price-level and induces a spurious negative relation between inflation and stock-returns. Chen, Roll, and Ross [6] control for the supply shock, but find that the relation remains significant.

(c) *Mundell-Tobin, and "shoe-leather" effects*: Expected inflation may induce a portfolio shift towards real assets and reduce their rate of return. It may also lead people to economize on their money holdings, a process that may entail real costs. Neither effect pertains to *unexpected* inflation, however, and yet the later, too, comoves negatively with real stock-returns.

1.2 *Monetary theory, and what this paper adds*

Modern monetary theory does not explain why stocks do worse in inflationary times . First, in the signal-confusion model of Lucas [17] there are no firms: Workers are self employed and there is no capital so that there is no distinction between wages and profits, and no stock market. Second, in a sequential service model such as [7], surprise inflation should *raise* firms' profits because it allows firms to sell more of their inventories. Finally, the "menu-cost" and nominal contracting literature could explain the phenomenon only by assuming that product prices are more rigid than wages, and this is implausible, especially in unionized industries.

Our explanation invokes an agency problem between firms and their workers. Firms and workers are temporarily confused between absolute and relative price changes. High nominal revenues signal that the worker has exerted effort, for which he should be rewarded. In other words, a positive price-level shock makes sellers think they are producing higher quality goods than they really are. They divide this apparent windfall with workers who get a higher real wage. Therefore money affects real activity by altering the distribution of income -- from shareholders to workers. The result hinges on the assumption that wages may be state-contingent (may depend both upon the sale price of the product and on the general price-level, that is eventually revealed), and that the wage contract must be renegotiation-proof.

Aside from proposing an explanation for why inflation is negatively related to stock-returns, the paper makes several theoretical contributions. First, it extends the partial equilibrium principal-agent analysis in [16] that assumes the principal to be risk neutral with respect to an *aggregate* shock to the price-level. If agents are risk-averse, the effect of aggregate shocks cannot generally be diversified away, and so the assumptions in [16] need justifying in a general equilibrium setting.

On another level, our paper adds to the literature on agency in general equilibrium ([2], [4], [20], [21], [22], by introducing nominal risk. In our structure, equilibrium is Pareto-optimal when the money-supply process is fully predictable. But an unpredictable money-supply rule strictly reduces welfare. The latter welfare result is stronger than the one proved in Lucas [17], and it derives from the requirement that the contract between the firm and the worker be renegotiation-proof.

Finally, ours is a signal-confusion model. In contrast to [17], however, ours recognizes that, while many goods markets indeed clear through impersonal spot trading, the market for labor services is, at least in advanced economies, overwhelmingly a contract-mediated market. Our model explains this feature of the world as well -- given that the firm cannot see the worker's effort and given that it takes time to see a signal on the worker's output (our two cornerstone assumptions), contracts naturally drive out spot market trades in the labor market, because such trades could not induce workers to exert effort.

1.3 *Plan of the paper*

Section 2 develops the model, defines an equilibrium and shows that one exists. Section 3 shows that random money-supply rules strictly reduce welfare. Section 4 shows that money injections have a real effect, and ends with a proof (in Theorem 4) of the claim that unanticipated inflation is negatively related to real stock-returns. Section 5 contains concluding remarks, and the Appendix contains the proofs of some of the claims made in the body of the paper.

2. The Model

We model a monetary economy populated by 2-period lived overlapping generations of agents. Because the assumed timing of events is important, we shall start with an overview of the model in terms of the sequence of events that occur during the course of a discrete period of time.

2.1 *The sequence of events within a time-period*

The economy has overlapping two-period-lived generations, and a single good -- “corn”. Only the old wish to consume. The young are endowed with K units of corn, a unit of labor, and no money. The old hold money (paid to them in the form of wages and stock market earnings in the previous period), and corn that they stored in their youth. All the money in this economy is held by people, not firms, from one period to the next. The precise sequence of events during the period is as follows:

a) The young decide how much corn to store, and how much of it to invest in the shares of firms,

b) The young sign labor contracts with firms,
c) The young chose effort,
d) The money balances of the old are multiplied by a random factor m (not yet revealed to the firms or to the young), and the old then spend all their money on output,
e) The old consume the goods they bought and the corn they stored, and die,
f) Each firm and its worker lean their own revenues, s, but not yet those of other firms,
g) Wage contracts might now be renegotiated,
h) The price-level, p, is revealed to all, and wages and dividends are paid,
i) Firms are liquidated and their corn perishes,
j) The young hold money (equal to their wages and the proceeds of their investment in stocks), and their stored corn into the next period.

The crucial item is f). If the labor contract could not be renegotiated at this stage, the wage schedule would fully reflect the two parties' preferences prevailing at stage b). With renegotiation, however, the wage schedule only partially reflects these preferences since at stage f), the worker's effort decision is a bygone and there is no incentive concern. Without the possibility to renegotiate, a contract that was fully indexed to the price-level would then be optimal, and surprise money would not then effect the real wage or the real returns to stocks.

The timing of events also guarantees that demand for money is strictly positive -- Earnings in youth arrive too late to be converted into corn and stored, and, hence, the only way to save them is in the form of money.

2.2 *Preferences, storage, and production*

Preferences: There are 2-period lived overlapping generations. Lifetime utility is $U(c) - e$, where c is the consumption of corn in efficiency units, which occurs in old age, and where $e \in \{0, 1\}$ is effort exerted when young.

Storage technology: The young are born with an endowment of $K > 1$ units of corn. A young person can do one of two things with K. He can store it, in which case he gets βK units of the consumption good in the next period. Or he can, at the beginning of the period, exchange it for shares that entitle him to dividends at the end of the period, which he can use to finance consumption in old age. Neither storage of corn, nor its exchange for the securities of firms, requires the use of money.

Production technology: Firms produce output using corn, k, and the effort of its hired worker.

The quantity of output is fixed at unity, but its quality, $y \geq 0$ can vary. If $k < 1$, the firm will produce no output at all, and for $k > 1$, the marginal product of corn is zero. So, the firm will want exactly 1 unit of corn. Also, the firm can hire at most one worker. The realization of y is uncertain, and has a distribution conditional on labor input, e, and corn input k given by the following distribution function: [NOTE FOR THE COPY EDITOR: SOME EQUATIONS ARE NOT NUMBERED, BUT NEVERTHELESS, THEY SHOULD BE DISPLAYED AND NOT RUN INTO THE TEXT]

$$F_e(y) = \text{Probability}\{\text{output quality} \leq y \mid \text{effort} = e, \text{ and } k \geq 1\}.$$

$F_0(.)$ and $F_1(.)$ are assumed to have strictly positive densities $f_0(.)$ and $f_1(.)$ on the same interval on the line. This ensures that e is not fully revealed by any y.

Two assumptions about the viability of production: First, assume that if workers do not exert effort, production is not viable:

$$\int y dF_0(y) < \beta. \qquad \text{(A1)}$$

This inequality implies that $U(\beta K)$, the utility a young agent could get if he stored his corn and consumed it in old age, exceeds $\int U[y + \beta(K-1)]dF_0(y)]$, which is the utility that a young agent would receive if he stored only $K - 1$ units of his corn and invested the remainder in the stocks of firms whose workers exerted zero effort. Assumption (A) therefore implies that firms must elicit high effort from their workers if they are to provide their shareholders with more utility than they would get by simply storing their corn. Thus, an equilibrium in effect requires that workers exert effort.

Second, assume that under high effort the production sector *is* viable; that is, that the average product of corn exceeds its return in storage (both measured in utility):

$$\int U[y + \beta(K-1)]dF_1(y) - 1 > U(\beta K) . \qquad \text{(A2)}$$

(A1) and (A2) imply that even if the individual incurred all the risk entailed in production, he would want to exert effort.

Finally, we normalize and measure the quality of output in units so that aggregate output, $\int y dF_1(y) \equiv Y = 1$.

2.3 *Money-supply, money-demand, and the price-level*

Money supply: At the start of a period, aggregate money supply is normalized to equal unity. The money supply rule consists of an injection of $m - 1$ nominal balances, distributed in proportion to existing holdings. Assume that m is i.i.d., with CDF $G(m)$, and with density $g(m)$. Since the total amount of money before the transfer is unity, after the shock, the supply of money is m.

Money demand: Only the old get these money injections. By the time the young receive their wages and dividends, it is too late to invest them in the storage technology. Moreover, money is the only means of saving earned income into old age. Hence, being proportional to existing money holdings, money injections are proportional to earned nominal income when young, and the constant of proportionality is m.

The price-level and the quantity theory equation: Let p denote the price of a unit of quality of output. We shall refer to p as the price-level. Aggregate nominal supply is $pY = p$ (since $Y = 1$). It must equal aggregate nominal demand, m, and so

$$p = m.$$

This is the quantity theory equation as it applies in our model.[1] Therefore, the gross *real* yield on money is exactly one, since the price of efficiency units increases in the next period by exactly the factor m by which each saver's money balances are also multiplied.

2.4 *Monitoring output-quality and the delay in observing p*

The market-clearing price per efficiency-unit of output is p. Initially, however, a firm and its worker learn only the price at which their *own* output sells, and not p. They will learn p only later in that period. The assumed delay seems reasonable since it takes time to reliably assemble aggregate price data. There also are parallel delays in the publication of the money supply figures and they are often substantially revised some months later.

If the firm and its worker could directly observe the quality, y, of their output, they could infer p. So, we assume, instead, that each seller sees only his own sales $s = py$, and cannot disentangle p from y. That is, neither the firm nor its worker can observe y. This assumption seems reasonable if y is assumed to represent the quality of a differentiated product, or a product whose characteristics are hard to enumerate and describe objectively -- say a personal computer, or the services of a company's CEO.[2]

If the sellers of output do not, at least for a short period of time, know the exact quality of the output they are selling, and if they only learn it precisely after they have observed p, how does p get determined? Does this require that a consumer can evaluate a product's quality better than its seller can? Not necessarily. What one needs to assume is that consumers can costlessly comparison-shop, and that they are able to rank products' *relative* qualities. As a result, all products will sell at a common price, p, per unit of quality

2.5 *The labor market*

Only the young have a labor endowment. In a spot market, a young worker would receive a fixed sum of money and he then would, as a result, have no incentive to exert effort. Hence the only viable market for labor is a contract market in which the "price" of a worker is a contract, $w(s, p)$, which specifies the nominal payment that the worker is to get as a function of the sales that he generates, s, and as a function of the money-supply shock, p. We assume that this contract can, and will, later be renegotiated if and only if the firm and the worker both agree to it.

We shall analyze only a symmetric equilibrium in which every firm offers the same contract. This assumption is natural because each firm has the same technology, and each is atomless and hence has no impact on the product and factor markets as a whole.

2.6 *The stock market*

The stock market is also a contract market in which, in exchange for a unit of corn delivered to it at the beginning of the period, the firm promises a nominal payment $\delta(s, p)$ at the end of the period if today its state turns out to be (s, p).

Firms: There are one-period-lived firms, that are liquidated at the end of the period. They have access to the production technology described above. The shares of these firms are traded at the beginning of the period. A firm requires exactly 1 unit of corn (that depreciates fully when the firm "dies" at the end of the period), and it can employ exactly one worker. Under full employment, the number of firms will equal the number of young agents, which also equals the number of investors, and the measure of each is unity. Suppose that a firm signs a wage contract $w(s, p)$ with its worker. The firm's dividend then is

$$\delta(s,\ p) \equiv s - w(s,\ p).$$

Of course, dividends will need to be such as to induce investors to hold the right fraction of safe and risky assets in their portfolios.

The portfolio decision: Stored corn yields a unit gross real return of β. A consumer stores the corn himself, and consumes the proceeds directly. Total investment, K, divides into R units in a risky asset and the remaining K - R units in the safe asset. If σ_j denotes the share of firm j's shares in the investor's risky assets, investor i's nominal assets at the end of the period will then be

$$R\int \sigma_j \delta(s_j,\ p)dj \quad + \quad p\beta(K - R) + w(s_i,\ p)$$

where $w(s_i,\ p)$ is the wage paid by the firm that investor i works for.

Let $f_e(y)$ be the density of the distribution $F_e(y)$. The density of s conditioned on p then is $p^{-1} f_e(s/p)$. The realizations of y are assumed to be independent across firms, and the dividend on the risky market portfolio in state p is

$$\delta(p) = \int [s - w(s,\ p)]\ f_1(s/p)\, p^{-1} ds\ , \tag{1}$$

given that each firm offers the same contract $w(\cdot)$.

Real income and real consumption: Let $c(s,\ p)$ be the worker's real consumption (in old age) if he ends up with sales s when the price-level is p, and his portfolio allocates equal weight to all firms. Then, if he chooses a risky allocation of R = 1 and a representative portfolio of all firms,[3]

$$c(s,\ p) = \beta(K - 1) + \frac{\delta(p) + w(s,\ p)}{p}\ . \tag{2}$$

Therefore, the worker's real consumption in old age is the same as his real income in youth. To see why this must be so, observe, in (2), that nominal payments $\delta(p) + w(s, p)$ received in youth are deflated by the price-level, p, prevailing then, and not by the price-level in the following period (which is when the worker's consumption actually takes place). This is because, if we let m' denote the money supply next period when the worker will consume, his post-transfer nominal resources would equal $m'[\delta(p) + w(s,\ p)]$, while the price-level would then be $p' =$

$m'p$. Hence, m' cancels.

A restriction on feasible portfolio choices: If his choice were unrestricted, the young person would not want to hold a representative portfolio of shares. In particular, if he could hold a nonzero fraction of his wealth in his own firm, the worker's portfolio choice and his effort choice would become linked: His effort decision would influence not only his wage payment, but also the returns from his portfolio. Indeed, a negative position in his own firm would give a worker an incentive to shirk! Because of the adverse consequences that such “insider trading” possibilities may have on incentives, we shall assume that firms rule such possibilities out: No worker is allowed to hold a nonzero fraction of his own company’s shares, but there is no restriction on the number of shares he may hold in other firms. Because every firm is of measure zero and all firms are ex-ante alike, this restriction means, effectively, that each worker will find it optimal to hold a diversified portfolio of shares in all firms, obtaining the same nominal return $\delta(p)$.[4]

State-contingent prices and the value of a firm: Firms are traded at the outset, among the young. The demand-price of a share in firm j is defined to be V_j in units of corn. Let

$$h_e(s, p) \equiv p^{-1} f_e(s/p) g(p)$$

be the joint density of s and p conditional on effort. This density describes equilibrium *ex-ante* beliefs about the state that the firm will find itself in. Our first proposition derives the standard asset pricing relation in this setting. The only twist is that, because p is an aggregate shock, the state-contingent price of consumption will depend on (the only) aggregate shock, and we shall denote it by $\theta(p)$.

<u>PROPOSITION 1</u> (THE VALUE OF A FIRM): The market value of firm j is

$$V_j = \beta^{-1} \int \theta(p) \Big\{ \int \Big[\delta(s, p)/p \Big] h_1(s, p) ds dp = \beta^{-1} E \Big\{ \theta(p) \, \delta(s_j, p)/p \Big\}$$

where the state-contingent price of consumption is defined to be

$$\theta(p) = \frac{\int U'[c(s, p)] \, h_1(s, p) ds}{\int U'[c(s, p')] \, h_1(s/p')g(p') ds d p'} \equiv \frac{E\{U'(.) | p\}}{E\{U'(.)\}}. \qquad (3)$$

<u>PROOF</u>: In the appendix. ■

Therefore, the state-contingent price of consumption simply equals the (normalized) expected marginal utility of income in state p.

The value of the market portfolio: If all firms follow the same $w(s, p)$ policy, the value of the market portfolio is

$$V = \beta^{-1} \int \theta(p)\,[\delta(p)/p]\, g(p)\, dp = \beta^{-1} E\Big\{ \theta(p)\, \delta(p)/p \Big\} . \qquad (4)$$

Equilibrium in the market for shares: The expression for V_j is the demand-price for a firm's shares as a function of the properties of the firm's dividend, as described by the distribution $h_1(s, p')$. In equilibrium, therefore, $V_j \geq 1$ -- otherwise the firm could not raise any corn and it could not produce any output.

We assume that a firm can not store corn and return it in the form of dividends or in the form of wages at the end of the period. Firms need corn only for productive purposes. This means that to attract capital (i.e., corn), firm j must pay dividends such that $E\Big\{ \theta(p)\ \delta(s_j, p)/p \Big\} \geq \beta$. This inequality is imposed as a constraint on problem (7) below; however, since the firm will not want to pay any more than it has to, this constraint will hold with equality. In equilibrium, then, $V_j = V = 1$. In equilibrium, then, $V_j = V = 1$. In equilibrium, then, $V_j = V = 1$.

Note two things about the stock market equilibrium. First, if p is not random ($p \equiv 1$), there is no aggregate risk, and no risk-premium. In that case $\theta(1) = 1$ and, hence, $V = 1$, so that (4) implies that $\delta(1) = \beta$, which now becomes a no-arbitrage condition.

Second, when p is random, each firm uses $\theta(p)$ to value its real profits in state p, because this is how its shareholders will value their dividend incomes in that state. Its dividends are distributed to the cross-section of workers. The formula for V_j is the familiar one: Firm j is worth more if $\delta(s_j, p)$ is positively correlated with the marginal utility of consumption. Also, in general, there is a risk-premium associated with the diversified portfolio, in that $\beta^{-1} E\{\delta(p)/p\} \neq 1$. As the proof of Proposition 1 shows, the "excess return", $\delta(p)/p - \beta$, satisfies the equation

$$\int \theta(p)\Big[\delta(p)/p - \beta\Big]\, g(p)\, dp = 0 .$$

2.7 *Renegotiation-Proof Contracts*

We shall allow for the possibility of a renegotiation of the initial contract to occur after s is observed, but before p is observed. The aim of this subsection is to provide a simple characterization of renegotiation-proof contracts.

Interim beliefs and preferences: After they see s, the two parties hold the beliefs:

$$g(p|s) = \frac{h_1(s, p)}{\int h_1(s, p')dp'}.$$

These "interim" beliefs give rise to a set of interim preferences. Given an arbitrary contract w(.), a worker who has sales s, but who does not yet know p will have an interim expected utility of

$$U^*(s; w) \equiv \int U\left[\delta(p)/p + \beta(K - 1) + w(s, p)/p\right] g(p|s)dp.$$

The firm's shareholders will, using the state-contingent price $\theta(p)$ as a weight on their dividends in state p, have the interim expected utility

$$V^*(s; w) \equiv \int \theta(p)\,[s/p - w(s, p)/p]\, g(p|s)dp.$$

Definition of a renegotiation-proof w(.): The following definition asserts that a renegotiation-proof contract must be Pareto-optimal for each s, given the parties' interim preferences:[5]

<u>DEFINITION 1</u> (RENEGOTIATION PROOF CONTRACTS): The contract w_0 is renegotiation-proof if, for all s, and for all $\pi \in [0, 1]$, $w_0(s, .)$ solves the maximization problem

$$\text{Max}_{w} \left\{\pi V^*(s; w) + (1 - \pi)U^*(s; w)\right\}$$

subject to

$$V^*(s, w) \geq V^*(s, w_0),$$

$$U^*(s, w) \geq U^*(s, w_0), \text{ and}$$

$$0 \leq w(s, p) \leq s \quad \text{for all } p.$$

The first two constraints demand that both parties agree to a renegotiation. As π varies between zero to one, we trace out the contract curve (in the Edgeworth box sense) between the utilities defined by $V^*(s, w_0)$ and $U^*(s, w_0)$ (this is true because since the problem is convex in w, the utility-possibility curve is concave to the origin). In particular, $\pi = 0$ transfers any "interim" rents to the worker, and $\pi = 1$ transfers them to the firm. The third constraint requires that wages and profits be nonnegative -- the latter is sometimes called a "limited liability constraint". If the maximization problem does not have a solution, then w_0 is not the maximum, and hence it is not renegotiation-proof.

A characterization of renegotiation proof contracts: Consider the maximization problem in Definition 1. Form the Lagrangean, with the multipliers $\upsilon(s)$ and $\chi(s)$ associated with the first two constraints:

$$\mathcal{L}(s) = \pi V^*(s; w) + (1 - \pi)U^*(s; w) + \upsilon(s)[V^*(s, w) - V^*(s, w_0)] + \chi(s)[U^*(s, w) - U^*(s, w_0)]$$
$$+ \int \xi_1(s, p)\,[s - w(s, p)]g(p|\, s)dp + \int \xi_2(s, p)w(s, p)g(p|\, s)dp.$$

The multipliers $\upsilon(s)$ and $\chi(s)$ are nonnegative because $\mathcal{L}(s)$ is decreasing in the constraint parameters $V^*(s, w_0)$, $U^*(s, w_0)$, and s. The multiplier $\xi_1(s, p)$ (attached to the limited liability constraint) is nonnegative when $s = w(s, p)$, and the multiplier $\xi_2(s, p)$ (attached to $w(s, p) \geq 0$ constraint) is nonnegative when $w(s, p) = 0$. This leads to the first-order condition that must hold for each (s, p). Differentiating with respect to $w(s, p)$,

$$- \xi(s, p) - [\pi + \upsilon(s)]\, \theta(p)/p + [1 - \pi + \chi(s)]U'(.)/p = 0,$$

where $\xi(s, p) \equiv \xi_1(s, p) - \xi_2(s, p)$. The second-order derivative with respect to w is everywhere negative because U (and therefore U*) is concave, while V* is linear . From the above, we get the following expression:

$$U'[c(s, p)] = \frac{p\, \xi(s, p) + [\pi + \upsilon(s)]\, \theta(p)}{[1 - \pi + \chi(s)]}. \qquad (5)$$

This condition allows us to represent, in a simpler way, the consumption under a renegotiation-proof contract.

<u>LEMMA 1</u> (CHARACTERIZATION OF RENEGOTIATION-PROOF CONTRACTS): A contract w is renegotiation-proof if and only if it affords the worker a consumption-level

$$c(s,\ p\,;\gamma) \equiv \max\Big\{\delta(p)/p + \beta(K\text{-}1),\ \ \min\{[s + \delta(p)]/p + \beta(K - 1),\ \ U'^{\,-1}[\theta(p)\gamma(s)]\}\Big\} \quad (6)$$

where the function $\gamma(.)$ is real-valued, and does not depend on p.

<u>PROOF</u>: *Necessity*: Solving (6) for w(.), we observe that if $\xi(s,\ p) = 0$, $c(s,\ p;\gamma) < [s + \delta(p)]/p + \beta(K - 1)$ and noting that π, $\upsilon(s)$, and $\chi(s)$ do not depend on p, shows that in this event $c(s,\ p;\gamma) = U'^{\,-1}[\theta(p)\gamma(s)]$. On the other hand, $\xi(s,\ p) > 0$ implies that the constraint binds, so that in this event, $c(s,\ p;\gamma) = [s + \delta(p)]/p + \beta(K - 1)$. Finally, if $\gamma(s)$ were such that $c(s,\ p;\gamma) < \delta(p)/p + \beta(K\text{-}1)$, wages would be negative.

Sufficiency: First, such a contract satisfies the limited liability condition. Second, it can not be bettered in the region of (s, p)-space on which it satisfies limited liability <u>strictly</u>. More specifically, for all (p, p', s) such that $\xi(s,\ p) = \xi(s,\ p') = 0$, the contract equates the ratio of the two parties marginal utilities of real income:

$$\frac{\theta(p)}{\theta(p')} = \frac{U'[c(s,\ p)]}{U'[c(s,\ p')]}\,.$$

Among contracts in which the worker's real compensation depends only on aggregate risk, this contract is Pareto-optimal -- it is on the contract curve. Third, it remains to be shown that the worker would reject any fair gamble that would transfer income from states in which $\xi(s,\ p) = 0$ to and from states in which $\xi(s,\ p) > 0$. But the worker would take such a gamble only if it transferred consumption from states in which $U'(c)$ is low, to states in which it is high. But it follows from (6) that his consumption is lowest (and hence his U' highest) in the states where $\xi(s,\ p) > 0$, and the firm can not transfer consumption to this state without violating the limited liability condition. ■

This result is critical for what follows. It says that the worker's real wage depends on *s*, a nominal variable. The wage does not depend directly on p.[6] Also, note that if $\theta(p)$ were constant (which it would be if the utility function were quadratic), $\gamma(s)$ is just the worker's marginal utility of consumption.

Since the worker and the shareholders are risk averse, neither group likes unnecessary risk. This is a consequence of the lemma that we have just proved:

COROLLARY TO LEMMA 1: Lotteries (i.e., random functions w) at the renegotiation stage are not renegotiation-proof.

PROOF: Suppose w_0 were a lottery, a function of s, p, and a random variable. Extending the definitions of V* and U* in the obvious way, the only way w_0 enters the maximization problem is though the scalars $V^*(s, w_0)$ and $U^*(s, w_0)$. Because the agent is risk averse, the constraint-set is still convex. Setting w_0 to its mean (conditional on (s, p)) makes the agent strictly better off without violating either rationality constraint. ■

2.8 *The ex-ante optimal contract*

We now formulate the firm's decision problem of drawing up the optimal renegotiation-proof contract. The firm uses contracts $w(s, p)$ to bid for workers under the added constraints that its shareholders get the market rate of return V, and the limited liability constraint. Omitting the j subscript, the firm, taking $\theta(p)$ and $\delta(p)$ as given, solves the problem of maximizing its worker's utility:

$$\text{Max}_{w} \left\{ \int U[c(s, p)]\, h_1(s, p)\, ds\, dp \;-\; e \right\}, \tag{7}$$

subject to *five* constraints, first, that its shareholders get at least the market rate of return,

$$\int \theta(p) \int [s/p \;-\; w(s, p)/p]\, h_1(s, p)\, ds\, dp \;\geq\; \beta,$$

second, to the incentive compatibility constraint that ensures effort is elicited,

$$\int\int U[c(s,\ p)]\{h_1(s,\ p) - h_0(s,\ p)\}\ ds\ d\,p\ -\ e\ \ \geq\ \ 0,$$

third, subject to the limited liability constraint,

$$s\ -\ w(s,\ p)\ \geq\ 0,$$

fourth, subject to a non-negative wage constraint, $w \geq 0$, and finally, subject to the constraint that the contract be renegotiation proof. [7] Here $c(s,\ p)$ is given by (2). Now in view of Lemma 1, we know that the non-negative wage constraint, the limited liability constraint, and the renegotiation-proofness constraint will be met if we substitute $c(s,\ p;\ \gamma)$ for $c(s,\ p)$ in the above problem and then let the firm choose $\gamma(s)$ instead of $w(.)$. After this is done, the Lagrangean is

$$\begin{aligned}\mathfrak{L} \quad = \quad & \int U[c(s,\ p;\ \gamma)]\ h_1(s,\ p)\ ds d\,p \\ & + \lambda \int \theta(\,p) \Big\{ \int [s/\,p + \ \beta(K - 1)\ + \delta(\,p)/\,p\ -\ c(s,\ p;\ \gamma)\ ]\ h_1(s,\ p)\ ds \Big\} d\,p \\ & + \mu \Big[\int U[c(s,\ p;\ \gamma)]\{h_1(s,\ p) - h_0(s,\ p)\}\ ds\ d\,p\ -\ e \Big] \qquad (8)\end{aligned}$$

The description in (8) of the optimal contracting problem is more manageable, and so we can now define equilibrium more parsimoniously and prove its existence.

2.9 *Equilibrium and its existence*

<u>DEFINITION OF EQUILIBRIUM</u>: <u>Equilibrium</u> consists of five functions: $\gamma(s)$, $\delta(\,p)$, $\theta(\,p)$, $w(s,\ p)$ and $c(s,\ p)$ that satisfy equations (1) - (3), (6) and the restrictions in the maximization problem (7).

Although some properties of any equilibrium will later on be established for the case of general utility functions, the existence result itself is here established only for the case in which utility is quadratic:

<u>THEOREM 1</u> (EXISTENCE OF EQUILIBRIUM): Suppose that

(i) The supports of F_0, F_1, and G are finite intervals, excluding zero.

(ii) f_1 and f_0 are Lipschitz functions.

(iii) $U(c) = a_0 + ac - (1/2b)c^2$, with $ab > y_{max} + \beta(K - 1)$

Then an equilibrium exists.

PROOF: In the appendix. ■

3. Welfare

Monetary variability imposes unnecessary risk on firms and workers. It reduces welfare because it garbles the information channel through which the worker's effort is evaluated. The result holds for general utility functions.

THEOREM 2 (RANDOM MONEY-SUPPLY RULES STRICTLY REDUCE WELFARE): When monetary policy is random, welfare is strictly less than in the case where money supply is constant (or deterministic).

PROOF: The standard principal agent problem without renegotiation is a concave programming problem, and the optimal (second-best) contract is deterministic. This is seen by taking the worker's utility level as the control. The second-best contract is also unique. Under the constant money supply rule, there is no delayed signal, p, and hence the second-best contract is renegotiation proof. In this contract, U' must depend only on y, that is, only on the ratio s/p. If an equilibrium contract under a random monetary rule were to yield the same utility, U' would have to depend only on the ratio s/p, because only then would the agent's real compensation not be a function of the random variable p. Now this would hold in (6) if limited liability were to bind for all states, but then there would be no dividends, and no shareholder would want to invest in the firm. Therefore $\xi(s, p) = 0$ on a set of (s, p) of positive f_1 - measure. On that set, however, (6) implies that $\theta(p)\gamma(s)$ must depend on s/p only, and non-trivially so in view of the incentive compatibility condition. We now derive a contradiction to this last statement. Eq. (3) says that θ is proportional to $E\{U'(.)|\ p\}$, and since the distribution of the ratio s/p is independent of p, $E\{U'(.)|\ p\}$ must be independent of p. Therefore, $\theta(p)$ must be constant. But then $\theta(p)\gamma(s)$ is independent of p, which can only be true if $\theta(p)\gamma(s)$ does not depend on s/p -- a contradiction. ■

The analyses in [15] and [23] help provide an intuitive explanation for this result. Their logic would say the following: If p is random, s is not a sufficient statistic for y. In this situation p adds information about y, and therefore it *too* should be used to allocate consumption to the agent (and yet our renegotiation proof contract *ignores* p in setting real consumption). On the other hand, if p were not random, s *would* be a sufficient statistic for *y*. There would then be no further informative signal about *y*, and the renegotiation proof contract would then be second-best.

Lucas [17], Theorem 5, proves that equilibrium under a predictable money supply rule is Pareto-optimal in the class of non-random allocations, i.e., allocations that depend on the real shock alone. Our Theorem 2 tells us more: Predictable money *strictly* dominates random money-supply rules.

4. Inflation and the distribution of income

4.1. *The nature of the monetary nonneutrality*

Money shocks do not change the level of output, Y, which is fixed at unity. Instead, they cause shifts in the distribution of income between capital and labor. This is our next result, and it, too, holds for general utility functions.

THEOREM 3 (SURPRISE MONEY HAS REAL EFFECTS): Changes in m alter the consumption of a subset of agents of positive measure.

PROOF: Suppose m were neutral. As in the proof of Theorem 2, the worker is risk-averse, and this warrants at least partial insurance. Therefore $\xi(s, p) = 0$ on a set of (s, p) of positive f_1 - measure. Neutral money implies that $\delta(p)/p$ is a constant, and the equilibrium contract would be of the form $w(s, p) = pW(s/p)$ for some function $W(.)$. Then from (6), this would imply that $\theta(p)\gamma(s)$ would be a function of the ratio s/p only. And since $\theta(p)$ is constant, this means that $\gamma(s)$ $[=\gamma(py)]$ is a constant, except on a set of zero f_1-measure. But then the incentive compatibility condition of problem (7) cannot be met. ■

4.2 *A reduced form for the share of labor*

Let $\delta^*(p) \equiv \delta(p)/p$ be the share of asset holders. Aggregate output is unity, and so labor's share is

$$1 - \delta^*(p) = \int [w(py, p)/p]dF_1(y). \qquad (9)$$

Subtracting $\delta(p)/p + \beta(K - 1)$ from $c(s, p; \gamma)$ in (6), real wages are

$$w(py, p)/p = \min\{y, U'^{-1}[\theta(p)\gamma(py)] - [\delta(p)/p + \beta(K-1)]\}. \qquad (10)$$

Substituting from (10) into (9) and subtracting $\delta^*(p)$ from both sides yields

$$1 = \int \min\left\{ y + \delta^*(p), \quad U'^{-1}[\theta(p)\gamma(py)] - \beta(K-1)\right\} dF_1(y) . \qquad (11)$$

This is an implicit function that relates δ^* to the product of θ and γ, and it forms the basis for our next result.

4.3 *Inflation and stock-returns*

Unexpected inflation is a rise in *p*. The following theorem spells out the conditions under which such a rise in *p* will reduce stockholders' share in income, δ^*, and with it, the real return on stocks.

<u>THEOREM 4</u> (INFLATION REDUCES RETURNS TO STOCKS): If

(a) The likelihood ratio $f_1(y)/f_0(y)$ is increasing in y, with a slope that is bounded away from zero,

(b) U(c) is quadratic as specified in Theorem 1,

(c) Var(p) is sufficiently small,

then the share of capital $\delta^*(p)$ is decreasing in p, and the share of labor, $1 - \delta^*(p)$, is increasing in p.

<u>PROOF</u>: The proof proceeds in five steps:

(i) Suppose that for each fixed y, $\theta(p)\gamma(py)$ is an increasing (decreasing) function of p. Since neither y nor $\beta(K-1)$ depend on p, for (11) to continue to hold when p changes, $\delta^*(p)$ must move in the opposite direction from $U'^{-1}[\theta(p)\gamma(py)]$. And since U'^{-1} is a decreasing function, this proves $\delta^*(p)$ is increasing (decreasing) in p as $\theta(p)\gamma(py)$ is an increasing (decreasing) function of p.

(ii) If U is quadratic, $\theta(p)$ is a constant. This is because U' is linear in c, so that in (3), $E\{U'[c(s, p)]| p\}$ just depends on mean consumption, which equals $Y + \beta(K-1)$ for all p.

(iii) From (i) and (ii), if $\gamma(s)$ is monotone increasing (decreasing), so is $\delta^*(p)$.

(iv) Rearrange eq. (b.3) of the appendix to get:

$$\gamma(s) = \frac{\lambda}{1 + \mu\left[1 - \tau_0(s)/\tau_1(s)\right]} \qquad (12)$$

where $\tau_e(s) \equiv \int h_e(s, p)dp$, for $e = 0, 1$. Neither λ nor μ depend on s, and they both are strictly positive because the constraints they refer to in problem (8) are binding. Therefore, $\gamma(s)$ is decreasing in s if the ratio $\tau_1(s)/\tau_0(s)$ is increasing, and vice-versa.

(v) As $\mathrm{Var}(p)$ gets small, $g(p)$ converges to a unit mass on $p = 1$, and $\tau_e(s)$ converges to $f_e(s)$. Condition (c) then guarantees that $\gamma(s)$ is a strictly increasing function.

The assertion now follows from claims (i) through (v). ■

4.4 *Inflation and real wages*

Unexpected inflation redistributes income from capital to labor: As the returns to stock fall with *p*, (9) implies that real wages must rise. Intuitively, a higher *p* makes it seem like real output is higher than it really is, and workers receive part of this gain. Moreover, the effect on wages is there because of the agency problem that, in turn, stems from the unobservability of *y*. This side-implication raises two questions: First, is unanticipated inflation positively related to real wages, and, second, is the effect stronger in those sectors where output is harder to observe and effort harder to monitor, such as, say, the nonmanufacturing sector?

Keane's (1993) study finds that the answer to both questions is a qualified “yes”. He finds that real wages for all workers tend to rise with both inflation and money growth in the 1966-82 sample period. Most of the point estimates are not significant, but they at least are of the right sign. Moreover, the non-manufacturing sector does indeed show a stronger relation than the manufacturing sector, which may explain why other studies, such as [5], find a negative relation between inflation and real wages among manufacturing workers covered by union contracts. On the other hand, Holland's [14] results do not support our wage results because in his data, unanticipated inflation seemed to *reduce* real wages, and even more so in the private business sector as a whole than in manufacturing.[8]

5. Conclusions

We have studied a monetary economy in which the labor market and stock markets are mediated by contracts. Labor contracting contains the friction induced by the possibility of a renegotiation after some time. This friction interacts with randomness in monetary policy so as to reduce welfare, and so as to redistribute income between workers and shareholders. In particular, surprise inflation reduces stock-returns, as evidence confirms. On the other hand, the proposition that surprise inflation should raise real wages gets at best lukewarm support from the data.

Because the action is taken before the money shock hits, aggregate output is constant, and the model does not give rise to a conventional Phillips curve. But, since it implies that price-level shocks will redistribute income towards workers, the model suggests that such shocks will affect the sectoral composition of output -- there should be a rise in the output of those goods that workers buy disproportionately, and a fall in the output of goods that shareholders favor.

Independently of its eventual fate as an explanation of the facts on inflation and different kinds of income, the model has the satisfying feature that money is not neutral and contracts are not fully indexed -- a feature that isn't assumed but *derived.*

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Appendix

<u>PROOF OF PROPOSITION 1</u>: Let R denote the size of the investor's risky portfolio, and $K-R$ the size of his safe portfolio. Suppose he puts a fraction $(1 - \varepsilon)$ of his risky investment in a diversified portfolio of all firms <u>other</u> than firm j, and the remainder in firm j. The number of shares of firm j that he buys is $\varepsilon R/V_j$, and if each other firm's price is V, his holdings of each other firm will equal $(1 - \varepsilon)R/V$. The dividend income from this portfolio is

$$[(1 - \varepsilon)R/V]\int \delta(s_J,\ p)dJ \ + \ [\varepsilon R/V_j]\delta(s_j,\ p) \ = \ [(1 - \varepsilon)R/V]\delta(\ p) \ + \ [\varepsilon R/V_j]\delta(s_j,\ p)\ .$$

The equality follows because by the law of large numbers,

$$\int \delta(s_J,\ p)dJ \ = \ \int \delta(s,\ p)\ (1/\ p)f_1(s/\ p)d\ p \ \equiv \ \delta(\ p).$$

The cost of this portfolio is just R. Investor i's expected utility then is

$$\int U(\ p^{-1}\{(1\text{-}\varepsilon)R\delta(\ p)/V + \varepsilon R\delta(s_j,\ p)/V_j \ + \ p\beta(K - R) \ + \ w(s_i,\ p)\})\ p^{-1}f_1(s_j/\ p)ds_j h_1(s_i,\ p)\ ds_i d\ p.$$

We now drop the subscript i. The first order condition for a maximum with respect to ε is:

$$[R/V_j]\int \ p^{-1}U'(.)\ \delta(s_j,\ p)\ \ p^{-1}f_1(s_j/\ p)ds_j\, h_1(s,\ p)\ dsd\ p \ - \ [R/V]\int U'(.)\delta(\ p)\ h_1(s,\ p)\ dsd\ p \ = \ 0$$

(because $\int \ p^{-1}f_1(s_j/\ p)ds_j = 1$ integrates out of the second term). Evaluating U(.) at $\varepsilon = 0$,

$$V_j\,/V \ = \ \frac{\int\{\int U'[c(s,\ p)]\ h_1(s,\ p)\ ds\}\{\int \ \delta(s_j,\ p)\ \ p^{-2}f_1(s_j/\ p)ds_j\}d\ p}{\int\int U'[c(s,\ p)]\delta(\ p)\ \ p^{-1}h_1(s,\ p)\ dsd\ p}\,.$$

In equilibrium, $\int \delta(s_j, p)\ \ p^{-1}f_1(s_j/\ p)ds_j \ = \ \delta(\ p)$, so that $V_j\,/V = 1$. Finally, the condition of optimality for R implies

$$\int U'[c(s,\ p)]\{\delta(\ p)/\ pV \ - \ \beta\}\ h_1(s,\ p)\ dsd\ p \ = \ 0,$$

and the claim follows. ■

<u>PROOF OF THEOREM 1</u>: By Lemma 3, when the utility function is quadratic, $\theta(\ p)$ is a constant. Problem (7) now no longer depends on $\theta(\ p)$, as the latter is proportional to $1/\ p$. The proof has five steps. The first four steps deal with problem (7). They show that a solution for γ (a) exists, (b) is unique, (c) is in a Lipschitz family with a constant that does not depend on δ, and (d) is continuous in δ. Step (e) shows that the reverse map, from γ to δ is continuous, shows that the composite mapping $\gamma \to \delta \to \gamma$ is continuous, and applies a fixed-point theorem.

We now describe each step in detail.

(a) Existence of optimal γ. $\gamma(s)$ is the marginal utility of consumption. Since p and y are bounded, so is s. Therefore the control $\gamma(s)$ maps a bounded interval in R into R. The γ are bounded uniformly. From below, it is bounded away from zero by the inequality in (iii) which, in view of the limited liability condition, makes it impossible for consumption ever to reach its bliss level, ab (the level of consumption at which marginal utility is zero). Functions γ that fall below this value can be replaced by other functions that give the parties the same payoffs. By a symmetric argument, we can bound $\gamma(s)$ from above as follows: Since $K > 1$, consumption is bounded from below by the payoff from private storage. The definition of $c(s, p; \gamma)$ does not allow the firm to affect the worker's consumption through negative wages even if it should choose a function γ that would imply a negative wage. Therefore, neither the criterion nor the constraints are affected by any γ that violate this constraint. Therefore we know that if a maximal γ exists, its range will lie in a compact subset of the line. Next, we constrain γ to a family of Lipschitz functions, and show [in (c) below] that this constraint does not bind at the optimum. With the sup norm $\|.\|$ on functions γ, the space of controls is then compact. The two constraints confine the controls to closed subsets of this compact set, and therefore the set of admissible controls is compact. The criterion is clearly continuous in γ. Therefore, by Weierstrass's theorem, a maximal γ exists.

(b) Uniqueness of optimal γ. Because of the incentive compatibility constraint, we need a theorem that does not require convexity of the control set -- Theorem 2 of [18, p. 221]. Every assumption in that theorem is met here. We have two real-valued constraints, so the dual space is R^2. The Lagrangean (8) is Frechet differentiable in γ, and γ affects it only through $c(s, p; \gamma)$, which on the unconstrained region equals $U'^{-1}(\gamma(s)) = b[a - \gamma(s)]$. Applying the chain rule, the first-order condition is that for all s,

$$- \left\{ \tau_1(s) + \mu [\tau_1(s) - \tau_0(s)] \right\} \gamma(s) + \lambda \tau_1(s) = 0, \tag{b.1}$$

where $\tau_e(s) \equiv \int h_e(s, p)d p$. The second order condition is

$$\tau_1(s) + \mu [\tau_1(s) - \tau_0(s)] > 0 . \tag{b.2}$$

Because the integral is a sum, (b.2) ensures the global concavity of the Lagrangean in γ at the point (λ, μ). From (b.1)

$$\gamma(s) = \lambda \tau_1(s) / \{ \tau_1(s) + \mu [\tau_1(s) - \tau_0(s)] \} . \tag{b.3}$$

The numerator of (b.3) is positive. The denominator coincides with the left-hand side of (b.2). Since $\gamma(s)$ must be strictly positive, (b.2) is met. Thus we have shown that a unique maximum of the Lagrangean exists for the control $\gamma(s)$ for (λ, μ) fixed.

To show that this is also a unique maximum for the problem (7), one still has to show that (λ, μ) are unique. First, if λ is given, then μ will be uniquely defined. The reason is as follows: both constraints must bind at the optimum. If there were more than one value of μ, (b.3) shows that the incentive compatibility condition would not bind at one of these (divide numerator and denominator by $\tau_1(s)$, and observe that $d\gamma/ds$ is increasing in μ). So the incentive compatibility condition could not bind at more than a single μ for a given λ. So conditional on λ, μ is unique, and it is enough to show that λ is unique. From [18, p. 222], Theorem 1) λ is the derivative of the maximized criterion with respect to β if this derivative exists. Since $\theta(p) = 1$, since on the unconstrained region $-w(s, p) = \beta(K-1) + \delta(p)/p - c(s, p; \gamma)$, and since on that region $c(s, p; \gamma) = b[a - \gamma(s)]$, the constraint reads

$$\int \{s + \beta(K-1) + \delta(p)/p - b[a - \gamma(s)]\}\, h_1(s, p)\, ds\, dp \geq \beta,$$

where the domain of integration is (s, p) for which $c(s, p) = U'^{-1}(\gamma(s))$. Let γ_β be the optimal policy at β, and $v(\beta, \gamma)$ the criterion evaluated at the policy γ. Let $v(\beta)$ denote the maximized criterion, i.e., $v(\beta) = v(\beta, \gamma_\beta)$. When β shifts up by an amount Δ, the shareholders' constraint will be met if a new policy γ_Δ is put in place under which $\int b[\gamma_\Delta(s) - \gamma(s)]\}\, h_1(s, p)\, ds\, dp = \Delta$. To attain such a transfer to the shareholders, a feasible policy that does not disturb the incentive-compatibility constraint is one that transfers a constant marginal utility of income. Since the marginal utility of income in state s is $\gamma(s)$, the worker must give up $D/\gamma(s)$ units of consumption in state s to give up D utils in that state. We are about to define these policies in terms of first order derivatives, so that, for fixed $\Delta > 0$, there will be an error that (because $U(.)$ is analytic) is of order $O(\Delta^2)$. Consider the policies $\gamma_\Delta(s)^+ = \gamma_\beta(s) + D^+(\Delta)/\gamma_\beta(s)$, and $\gamma_\Delta(s)^- = \gamma_{\beta+\Delta}(s) - D^-(\Delta)/\gamma_{\beta+\Delta}(s)$ where $D^+(\Delta) = \Delta / \int [b/\gamma_\beta(s)] h_1(s, p)\, ds\, dp$. and $D^-(\Delta) = \Delta / \int [b/\gamma_{\beta+\Delta}(s)] h_1(s, p)\, ds\, dp$. The policy $\gamma_\Delta(s)^+$ is feasible at $\beta + \Delta$, which means that (ignoring terms of order Δ^2),

$$v(\beta + \Delta) \geq v(\beta, \gamma_\Delta^+) = v(\beta) - D^+(\Delta),$$

and and the policy $\gamma_\Delta(s)^-$ is feasible at β. This means that

$$v(\beta) \geq v(\beta, \gamma_\Delta^+) = v(\beta) + D^-(\Delta).$$

Therefore

$$\frac{-D^+(\Delta)}{\Delta} \;\leq\; \frac{v(\beta + \Delta) \; - \; v(\beta)}{\Delta} \;\leq\; \frac{-D^-(\Delta)}{\Delta}.$$

We substitute from the definition of $D(.)$ to conclude that the right-hand side and the left-hand side of the above pair of inequalities must converge to the same well-defined limit as long as $\int[1/\gamma_\beta(s)]h_1(s, p)dsd\,p$ is continuous in β. And using the functional form in (b.3), this could fail only if the criterion itself were discontinuous in β. But this would contradict Berge's Theorem of the maximum.

(c) <u>A uniform Lipschitz constant</u>. (b.3) implies $\gamma(s) = \lambda / \{1 + \mu[1 - \tau_0(s)/\tau_1(s)]\}$. Since $\gamma(s)$ is bounded uniformly in $[s, \delta(.)]$ and since λ and μ do not depend on s, it follows that λ and λ/μ are finite. Then it is sufficient that $\tau_1(s)/\tau_0(s)$ be Lipschitz. Now $\tau_e(s) = \int (1/p)f_e(s/p)g(p)$. Since by (i) p is bounded away from zero, and by (ii) f_1 and f_0 are Lipschitz. So $\gamma(s)$ is Lipschitz with a constant not depending on $\delta(.)$.

(d) <u>Continuity of γ in δ and θ</u>: We use the sup norm $\|.\|$ on δ as well. Since γ is in a Lipschitz family, $\gamma_1(s) \to \gamma_2(s)$ pointwise in s is equivalent to $\|\gamma_1 - \gamma_2\| \to 0$. So we only need to show that $\gamma(s)$ is continuous in δ at each s. From (b.3), this follows if λ and μ are continuous. Now the criterion and the constraints are continuous in δ, and so by Berge's theorem of the maximum, the set of maximizing values for $\gamma(s)$ is u.h.c. in δ, since by (b) above it is unique, it must be continuous.

(e) <u>Existence of equilibrium γ</u>: We use Schauder's fixed point theorem, as stated in Theorem 17.4 of [24]. Lipschitz families are equicontinuous. The set of bounded functions γ discussed in (a) is convex. With the sup norm, δ [defined in terms of γ by (6) (2) and (1)] is continuous in γ. That is, given γ, equation (6) yields c, which is then used in (2) and (1) in that order, to get values for w and then δ. Given this value for δ the problem (8) then yields a new solution for γ. This defines a continuous map from γ-space into itself that goes in the order: $\gamma \to w \to \delta \to \gamma$. Schauder's theorem then gives us a fixed point. ■

Endnotes:

1. In terms of the quantity-theory equation $m\text{V} = p\text{Y}$, in our model, velocity, V = 1, and we have normalized Y = 1. Therefore $m = p$.

2. In practice, such uncertainty appears to exist in most markets -- most experts now think that, as it is computed at present, the Consumer Price Index does not sufficiently allow for the growth in the quality of goods and services, and that as a result, it grows faster than it should [11]. Moreover, experts also disagree abut what the precise rate of output's quality-growth really is, which strongly suggests that no one really knows what the “true” rate of inflation actually is.

3. Since they need exactly one unit of corn to operate, and since the measure of all firms is 1, the total equity (i.e., “risky”) investment by each young person will, in equilibrium, equal R = 1.

4. Bisin and Guaitoli [4] call a contract *exclusive* if its terms depend on how a party to it deals with some other parties in the economy. Our wage contract is exclusive in this sense because it does not allow the worker to deal in the shares of his own firm.

5. Our renegotiation-proof allocation is interim Pareto-optimal and unique given <u>any</u> π, so there is no renegotiation process under which both parties would agree to change this allocation. Maskin and Tirole [19] formalized this notion as *strong renegotiation proofness*.

6. The timing of renegotiation is important for the conclusions. For instance, if
(a) p were revealed at the same time as s (which is empirically implausible, however) or alternatively if renegotiation were impossible before p was revealed (equally implausible given the lengthy delays in observing the CPI) there would be no real effect of p. Or if
(b) The parties could renegotiate after the choice of effort, but *before* they learn s, the real effects of p would again be eliminated; they would renegotiate to make the real wage independent of s.

7. This contracting problem relates to a sizeable literature on contracts as a risk-sharing device when markets are incomplete. For instance, in [13], bank deposits soften the impact of liquidity shocks on individuals. And in [3], commodity futures contracts improve risk sharing because the suppliers of commodities are risk-neutral. As is typical in this literature, these papers assume that the shocks are unverifiable. In contrast, our shocks (s and p) are verifiable at least with a delay, and hence a complete contract is feasible in our context. Indeed, a major point of our paper is that it shows why feasible, fully-indexed contracts are not used in equilibrium.

8. We discuss this and other evidence further in [16].